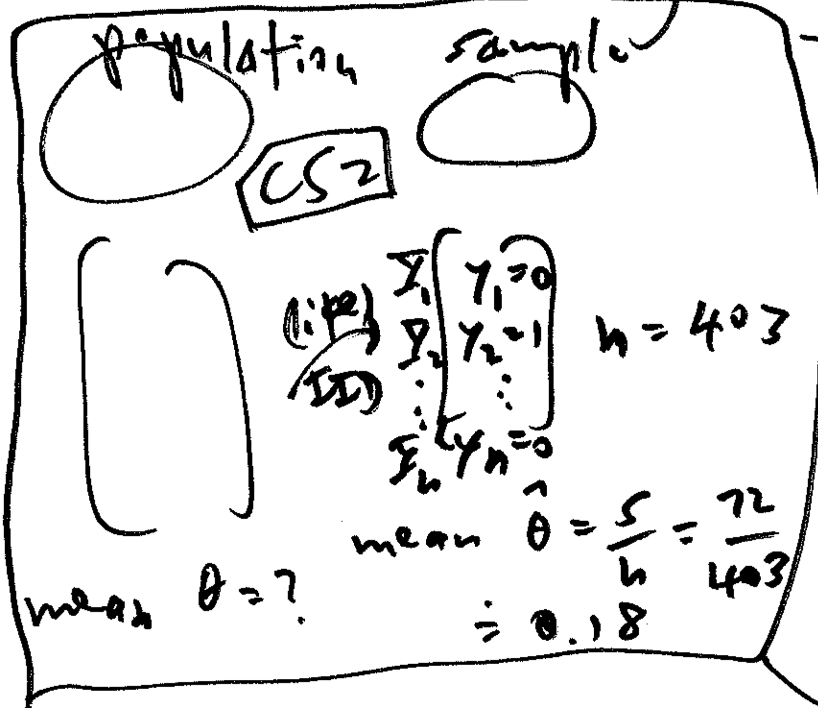
Notes on Neyman-style frequentist inference

① (large-sample version) The diagram with the 3 datasets (population, sample, repeated-sampling) completely describes frequentist inference (Neyman-style) but is cumbersome.

Remarkably, the entire diagram can be reduced to a single line of math:



$$(Y_i \mid \theta\, \mathcal{B}) \stackrel{IID}{\sim} \text{Bernoulli}(\theta) \quad (i = 1, \dots, n) \qquad (*)$$

from which $\hat{\theta} = \frac{S}{n}$, (random variable)

$$S = \sum_{i=1}^{n} Y_i$$

Once you've decided to use $\hat{\theta}$ as your estimate of θ, (*) can be reduced even further in this case study: Under sampling model (*), $(S \mid \theta\, \mathcal{B}) \sim \text{Binomial}(n, \theta)$

Later in the course θ $\leftarrow$ parameter will be a vector of length ≥ 1, $\theta = (\theta_1, \dots, \theta_k)$, $\theta_j \in \mathbb{R}$. But for now, $k = 1$

Sampling model (B) is a special case of the following:

$$\boxed{\begin{array}{c}(Y_i \mid \theta \, \mathcal{B}) \overset{\text{IID}}{\sim} p(y_i \mid \theta \, \mathcal{B}) \\ (i = 1, \dots, n) \qquad \theta \in \mathbb{R}\end{array}} \quad (**)$$

In which

$p(y_i \mid \theta \, \mathcal{B})$ is a family of parametric distributions (PMF discrete, PDF continuous) indexed by θ.

Algorithm for large-sample (big n) Neyman-style frequentist inference

(1) Specify $p(y_i \mid \theta \, \mathcal{B})$ in a manner appropriate to the context $\mathcal{C}$ of the problem $\mathbb{P}$.

(2) Choose a good estimate $\hat{\theta} = h(y_1, \dots, y_n)$;

good = (intuitively captures most or all of the information in $y = (y_1, \dots, y_n)$ about θ; obeys a Central Limit Theorem).

③ work out the details of the CLT for $\hat{\theta}$: for large n, $(\hat{\theta} \mid \theta B) \underset{RS}{\overset{\cdot}{\sim}} N\left(\theta, \frac{V(\theta)}{n}\right)$

$\boxed{\text{RS = repeated sampling}}$

(need to be able to figure out $V(\theta)$ and how to estimate it)

④ use this to define

$$\hat{SE}(\hat{\theta}) = \sqrt{\frac{\hat{V}}{n}}.$$

⑤ Approximate $100(1-\alpha)\%$ CI for θ is then

$$\hat{\theta} \pm \Phi^{-1}\left(1-\frac{\alpha}{2}\right) \cdot \hat{SE}(\hat{\theta}).$$