MCMC

AMS206 5 Mar 19

Random-effects Poisson regression

	θ_1	θ_2	θ_3	γ
1	θ_{11}^*			
2				
⋮				
M				
mean	$\bar{\theta}_1^*$	$\bar{\theta}_2^*$	$\bar{\theta}_3^*$	$\bar{\gamma}^*$
SD	s_{θ_1}	s_{θ_2}	s_{θ_3}	s_γ

(IID) ① MC data set

$$\widehat{MCSE}(\bar{\theta}_j^*) = \frac{\hat{\sigma}_{\theta_j}}{\sqrt{M}} = \frac{s_{\theta_j}}{\sqrt{M}} = \hat{T}$$

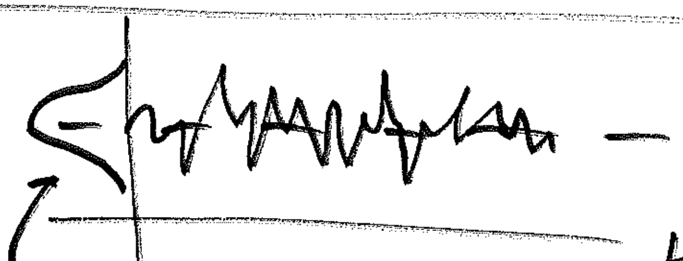
$$\rightarrow \hat{M} = \left(\frac{s_{\theta_j}}{\hat{T}} \right)^2$$

0.0005 $\hat{M}$

→ 0.00005 $100\hat{M}$

(t) time	μ_t^*
1	
2	
3	
⋮	

$p(\mu \mid y \ldots \mathcal{B})$

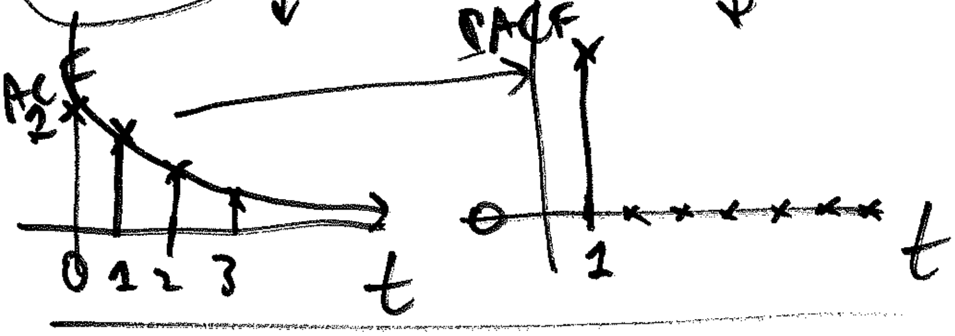


t (iteration #)

$$\mu^*_t = \beta_0 + \beta_1 \mu^*_{t-1} + e_t$$ ← log 1

$e_t \overset{IID}{\sim} N(0, \sigma_e^2)$

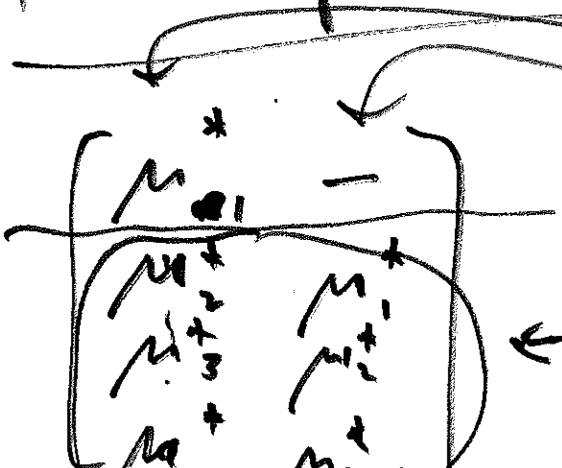
1st order autoregressive (AR_1) model



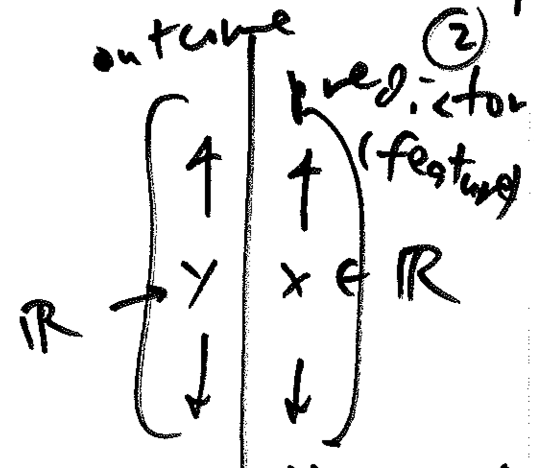
IID ~ $N(0, \sigma_e^2)$ ↔ "white noise"

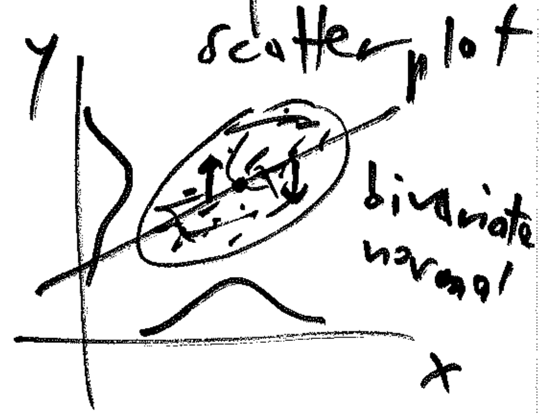
ACF = autocorrelation function

PACF = partial ACF



← cor(mu.star, mu.star.logged)





$$y_i = \beta_0 + \beta_1 x_i + e_i$$

$e_i \overset{IID}{\sim} N(0, \sigma_e^2)$

(simple linear regression)

if $\mu_t^* \sim AR_1(\hat{\rho})$ & $\bar{\mu}^* = \frac{1}{m}\sum_{t=1}^{m} \mu_t^*$

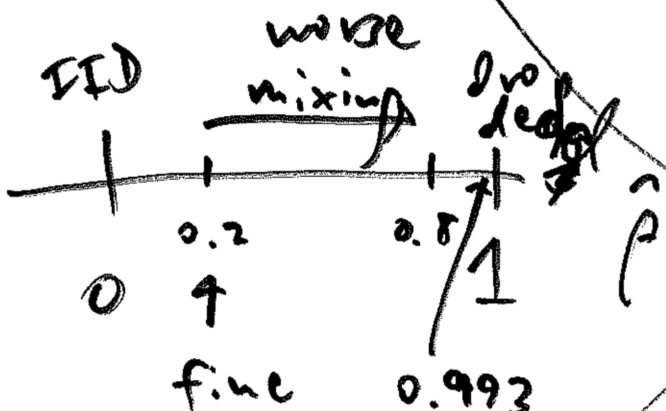
first-order autocorrelation

Galton/Pearson (r)

then $\widehat{MCSE}(\bar{\mu}^*) = \frac{s_\mu}{\sqrt{m}} \sqrt{\frac{1+\hat{\rho}}{1-\hat{\rho}}}$

↓

$+\infty$ as $\hat{\rho} \uparrow 1$



$\underset{\sim}{\theta} = (\mu, \sigma, \nu)$

$(y_i | \underset{\sim}{\theta}\, \mathcal{B}) \stackrel{IID}{\sim} t_\nu(\mu, \sigma)$

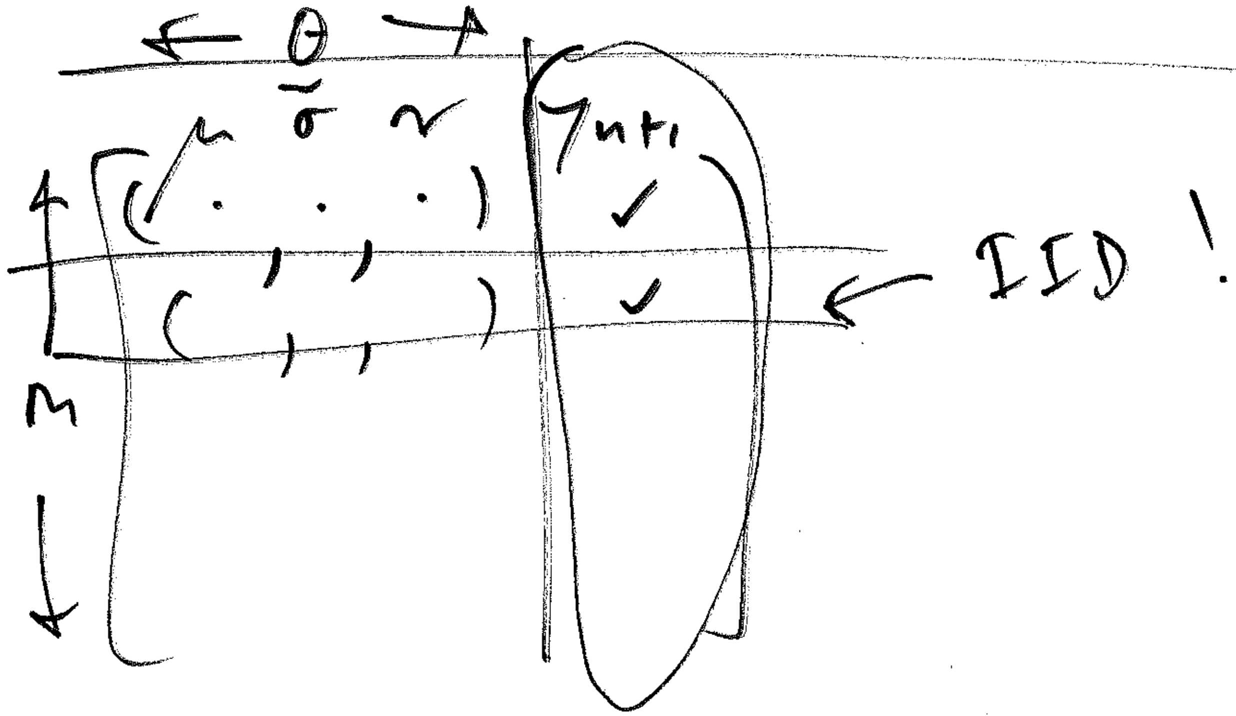
$\int\int\int_{\mu\,\sigma\,\nu}$ (LTP)

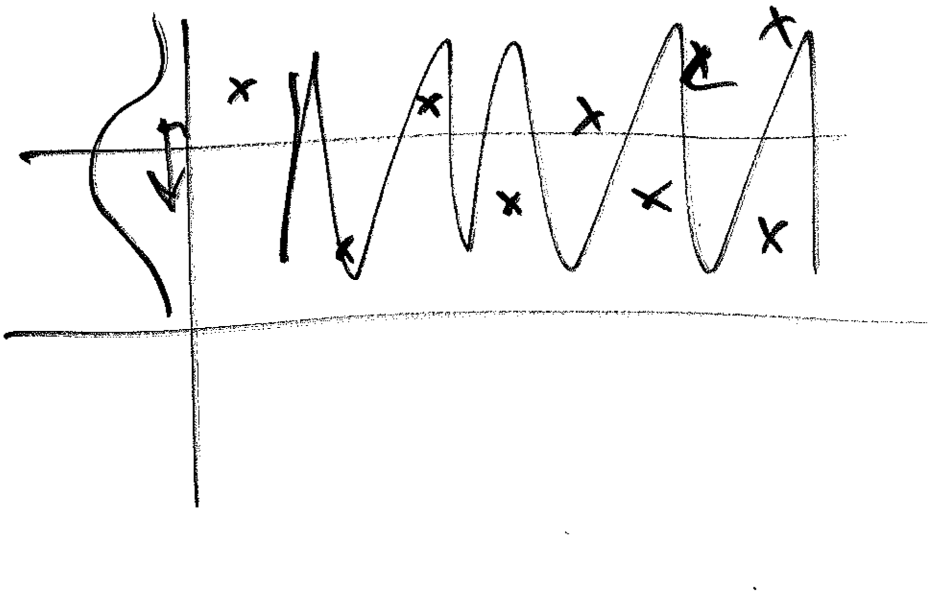
$$p(y_{n+1} | \underset{\sim}{y} \ldots \mathcal{B}) = \int_{\Theta} p(y_{n+1}\, \underset{\sim}{\theta} \,|\, \underset{\sim}{y} \ldots \mathcal{B})\, d\underset{\sim}{\theta}$$

$$= \int_{\Theta} \underbrace{p(y_{n+1} | \underset{\sim}{\theta}\, \cancel{y} \ldots \mathcal{B})}_{t_\nu(\mu,\sigma)\ \text{sampling dist.}} \cdot \boxed{p(\underset{\sim}{\theta} | \underset{\sim}{y} \ldots \mathcal{B})}\, d\underset{\sim}{\theta}$$

posterior for $\underset{\sim}{\theta}$

$$(y_{n+1} \mid \underset{\sim}{y}) \longleftrightarrow \left\{ \begin{array}{l} (\underset{\sim}{\theta} \mid y) \\ (y_{n+1} \mid \underset{\sim}{\theta}, \not{y}) \end{array} \right\}$$





$(C_i \mid \lambda_C \, \mathbb{P} \, \mathcal{B}) \overset{IID}{\sim} \text{Poisson}(\underline{\underline{\lambda_C}})$ ← Poisson

$i = 1, \ldots, n_C$

↕ indep

$(E_j \mid \lambda_E \, \mathbb{P} \, \mathcal{B}) \overset{IID}{\sim} \text{Poisson}(\lambda_E)$

$j = 1, \ldots, n_E$

$\lambda_C - \lambda_E$

$$\frac{\lambda_E - \lambda_C}{\lambda_C}$$
