logistic regression

(ML & Bayes)

AMS 206 19 Mar 19

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$$(Y_i \mid p_i\, B) \stackrel{I}{\sim} \text{Bernoulli}(p_i) \quad \boxed{1} \quad (i = 1, \dots, n)$$

pregnancies, age

$$\text{logit}(p_i) = \log\left(\frac{p_i}{1 - p_i}\right) = \underbrace{\beta_0 + \beta_1 x_{i1} + \dots + \beta_{k-1} x_{i\,k-1}}_{\sum_{j=1}^{k} \beta_j x_{ij}} \quad \boxed{2}$$

this is a special case of generalized linear models (GLMs): Nelder & McCulloch (1982)

$$\boxed{2} \quad p_i = F\left(\sum_{j=1}^{k} \beta_j x_{ij}\right)$$

continuous CDF

link function

$$\leftrightarrow \boxed{2'} \quad F^{-1}(p_i) = \sum_{j=1}^{k} \beta_j x_{ij}$$

$$F^{-1}(p_i) = \log\left(\frac{p_i}{1 - p_i}\right) \leftrightarrow F \sim \text{logistic distribution}$$

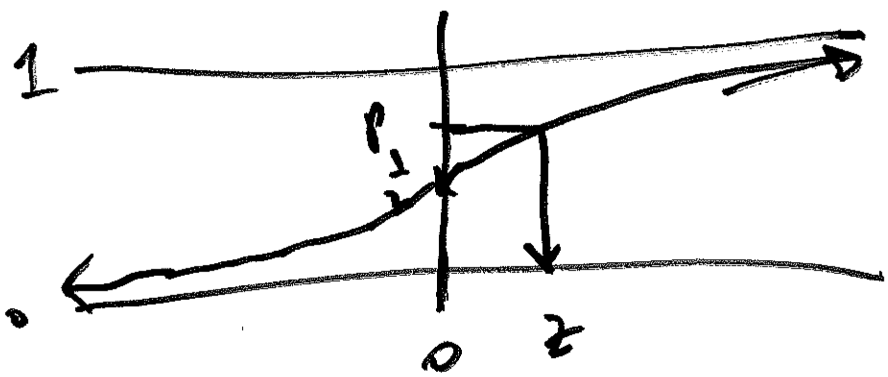
$\boxed{pr}$ $\boxed{2}$

inverse CDF of $N(0,1)$

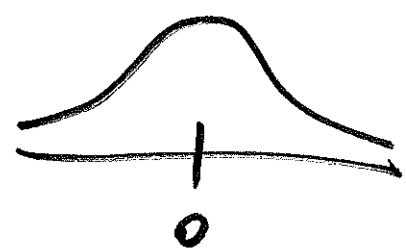
$$\Phi^{-1}(p_i) = \sum_{j=1}^{k} \beta_j x_{ij}$$

K. Pearson (1895) probit regression

$$\text{probit}(p_i) = \Phi^{-1}(p_i)$$

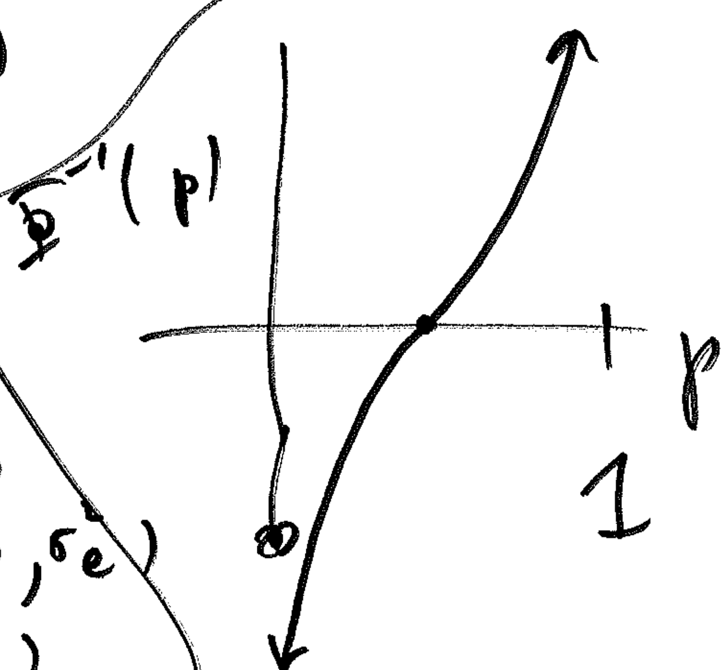


CDF of $N(0,1)$



$$\Phi(z) = p$$

i.e. $p = \Phi^{-1}(z)$



mult. lin. reg.

$$Y_i = (X\beta)_i + e_i$$

e_i IID $N(0, \sigma_e)$

$$\underset{\sim}{\theta} = (\beta_1, \ldots, \beta_k, \sigma_e^2)$$

$$\ell\ell(\underset{\sim}{\beta} \mid \underset{\sim}{y}\, X\, \mathcal{L}\, \mathcal{B}) = \sum_{i=1}^{n} \left[y_i \log \left\{ \frac{1}{1+\exp[-(X\underset{\sim}{\beta})_i]} \right\} + (1-y_i) \log \left\{ 1 - \frac{1}{1+\exp[-(X\underset{\sim}{\beta})_i]} \right\} \right]$$

$$\hat{\underset{\sim}{\beta}}_{MLE} = \underset{\underset{\sim}{\beta}}{\operatorname{argmax}}\; \ell\ell(\underset{\sim}{\beta} \mid \underset{\sim}{y}\, X\, \mathcal{L}\, \mathcal{B})$$

must be searched for numerically; efficient algorithm: <u>Fisher scoring</u> (Taylor series; Newton-Raphson)

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k large: <u>(stochastic) gradient descent</u> to $\underset{\underset{\sim}{\beta}}{\operatorname{argmin}} - \ell\ell$

$$(\beta \mid N^P) \sim N(\mu, \sigma^2)$$

$$p(\beta \mid N^P) = c \exp\left[-\frac{1}{2\sigma^2}(\beta - \mu)^2\right]$$

$$= c \exp\left[-\frac{1}{2}(\beta - \mu)^T \left(\frac{1}{\sigma^2}\right)(\beta - \mu)\right]$$