Bayes | soon: multiparameter problems | read: Gelman et al. ch. 3 | AMS 206 14 Feb 19

$\theta = (\theta_1, \dots, \theta_k)$, $\theta_j \in \mathbb{R}$

$$p(\theta \mid y\, \mathcal{B}) = c\; p(\theta \mid \mathcal{B})\; \ell(\theta \mid y\, \mathcal{B})$$

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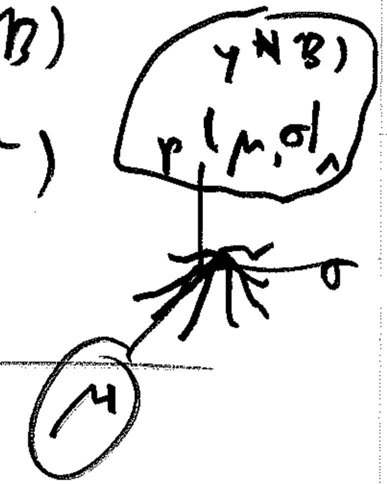
$$(\mu\, \sigma \mid \mathcal{B}) \sim p(\mu\, \sigma \mid \mathcal{B})$$

$$(Y_i \mid \mu\, \sigma\, \mathcal{B}\, \mathbb{N}) \overset{IID}{\sim} \underline{\underline{N}}(\mu, \sigma^2)$$

$$(i = 1, \dots, n)$$

normality assumption

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$$p(\mu \mid y\, \mathcal{B}\, \mathbb{N}) = \int_0^\infty p(\mu\, \sigma \mid y\, \mathcal{B}\, \mathbb{N})\, d\sigma$$

$$p(\sigma \mid y\, \mathcal{B}\, \mathbb{N}) = \int_{-\infty}^{\infty} p(\mu\, \sigma \mid y\, \mathcal{B}\, \mathbb{N})\, d\mu$$

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$\theta = (\theta_1, \dots, \theta_k)$, $k$ large

$$p(\theta_1 \mid y\, \mathcal{B}) = \int\!\!\int \cdots \int p(\theta_1, \dots, \theta_k \mid y\, \mathcal{B})\, d\theta_2\, d\theta_3 \cdots d\theta_k$$

Laplace approximation

$$(\theta \mid \mathcal{B}) \sim \text{Beta}(\alpha, \beta) \rightarrow E(\theta \mid \mathcal{B}) = \frac{\alpha}{\alpha+\beta}$$

(*)

posterior mean = $\underbrace{\ }_{n_0 = \text{prior sample size}}$ prior mean + data mean

$$\frac{\alpha+s}{\alpha+\beta+n} = \underbrace{\frac{\boxed{\alpha+\beta}}{\alpha+\beta+n}}_{w} \left(\frac{\alpha}{\alpha+\beta}\right) + \underbrace{\left(\frac{\boxed{n}}{\alpha+\beta+n}\right)}_{1-w} \left(\frac{s}{n}\right)$$

$0 \le w \le 1$

$$p(\theta \mid \mathcal{B}\,\mathbb{B}) = \text{Beta}(\alpha, \beta)$$

$$p(\theta \mid y\,\mathcal{B}\,\underbrace{\mathbb{B}}_{\text{Beta prior}}) = \text{Beta}(\alpha+s, \beta+n-s)$$

$$(y_1, \dots, y_n) = y$$

$$s = \sum_{i=1}^{n} y_i \qquad \bar{y} = \frac{s}{n}$$

(*) $V(\theta \mid \mathcal{B}) =$

$$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

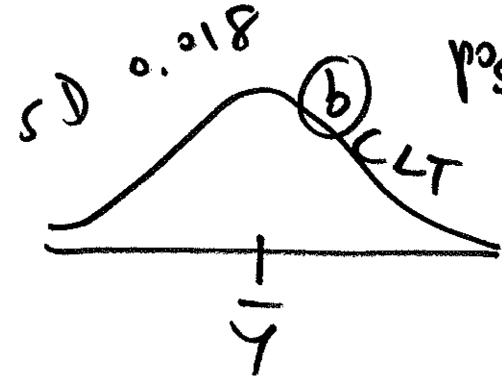
$$= \underbrace{\left(\frac{\alpha}{\alpha+\beta}\right)}_{\hat{\theta}_{prior}} \underbrace{\left(\frac{\beta}{\alpha+\beta}\right)}_{(1-\hat{\theta}_{prior})} \underbrace{\left(\frac{1}{\boxed{\alpha+\beta}+1}\right)}_{\frac{1}{n_0}}$$

prior variance $= \dfrac{\hat{\theta}_{prior}(1-\hat{\theta}_{prior})}{n_0}$

prior

(Neyman) F | B 3


$$c_1 e^{-c_2\left(\bar{Y}-\theta\right)^2}$$

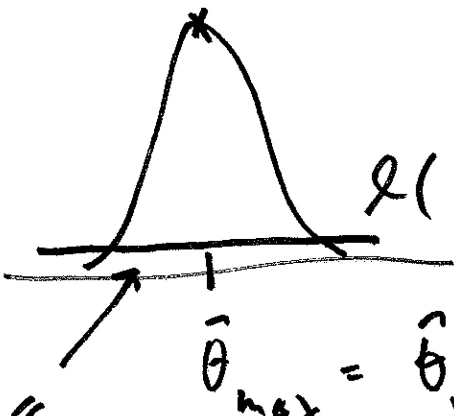



posterior dist for $\theta$ given $\bar{y}$ & (**)

$$c_1 e^{-c_2\left(\theta-\bar{y}\right)^2}$$

n large, not much prior info (**)

# Bernstein – von Mises Theorem

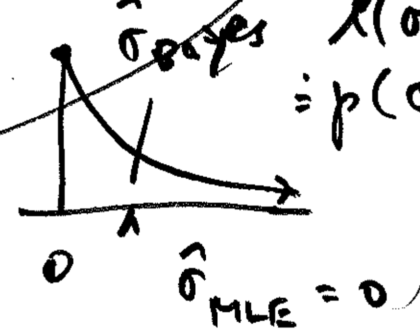



n large, not much prior info

$$\ell(\theta \mid y\, \mathcal{B}) \doteq p(\theta \mid y\, \mathcal{B})$$

$\in \mathbb{R}$

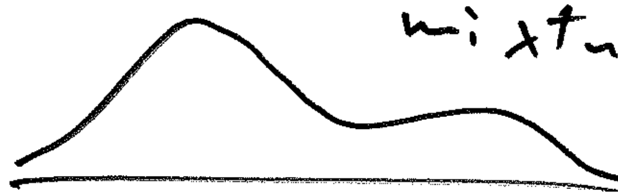
diffuse prior = flat prior




$\ell(\sigma \mid y\, \mathcal{B}) \doteq p(\sigma \mid y\, \mathcal{B})$

$N(\mu, \sigma^2)$

mixture



$p\, N(\mu_1, \sigma_1^2) + (1-p)\, N(\mu_2, \sigma_2^2)$